Exam Advanced Logic

June 14th, 2016

Instructions:

- Put your student number on the first page and subsequent pages (not your name, in order to allow for anonymous grading).
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- Your exam grade is computed as min(10, (the sum of all your points + 10) divided by 10). For example, someone who would only fill in a student number would get a 1, while someone who would make questions 1-9 perfectly but would skip the bonus question would get min(10, (90+10)/10) = 10.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Rineke Verbrugge.
- Please fill in the anonymous course evaluation.

Good luck!

1. **Induction (10 pt)** Consider the sublanguage $\mathscr{L}_{\leftrightarrow}$ of the language of propositional logic with $\leftrightarrow$ as its only logical operator. (So without $\neg$, $\wedge$, $\vee$ and $\rightarrow$).
   (a) Give an inductive definition of the well-formed formulas of $\mathscr{L}_{\leftrightarrow}$.
   (b) Prove by induction that each well-formed formula of $\mathscr{L}_{\leftrightarrow}$ is satisfiable.
   (Reminder: A well-formed formula A is satisfiable if and only if there exists a valuation $v : P \rightarrow \{0, 1\}$ such that $v(A) = 1$.)

2. **Three-valued logics (10 pt)** Using a truth table, determine whether the following inference holds in $\mathbf{K}_3$:

$$(p \wedge q) \wedge \neg p \models_{K_3} (p \vee q) \supset \neg p$$

   Do not forget to draw a conclusion.

3. **Tableaus for FDE and related many-valued logics (10 pt)** By constructing a suitable tableau, determine whether the following inference is valid in $\mathbf{Ł_3}$. If the inference is invalid, provide a counter-model.

$$(p \supset q) \wedge (r \supset s) \vdash_{Ł_3} (p \supset s) \vee (r \supset q)$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in the fuzzy logic with $D = \{x : x \geq 0.7\}$. If so, explain why. If not, provide a counter-model and explain why not.

$$\models_{0.7} ((p \rightarrow q) \rightarrow q) \rightarrow q$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

$$\Diamond(p \wedge \Diamond(q \wedge \Diamond p)) \vee (p \wedge \Diamond(q \wedge \Diamond\Diamond p)) \vdash \Diamond\Diamond\Diamond p$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following tense-logical inference is valid in $K^t_{\eta\tau}$. If the inference is invalid, provide a counter-model.

$$[F]([F]p \supset p) \vdash_{K^t_{\eta\tau}} [F]p$$

NB: Do not forget to draw a conclusion from the tableau.

7. **Soundness and completeness (10pt)** Let b be a complete open branch of a $K_{\tau\sigma}$-tableau, and let $I = \langle W, R, v \rangle$ be the interpretation that is *induced* by b. Show that the accessibility relation R of I is transitive and symmetric.

8. **First-order modal tableau, variable domain (10 pt)** By constructing a suitable tableau, determine whether the following is valid in VK. If the inference is invalid, provide a counter-model.

$$\forall x Qx \supset \forall x \Box Px \vdash_{VK} \forall x Qx \supset \Box \forall x Px$$

NB: Do not forget to draw a conclusion from the tableau.

9. **Default logic (10 pt)** The following translation key is given:

$P(x)$	x is a fan of Graham Priest
$B(x)$	x is a fan of Ruth Barcan-Marcus
$K(x)$	x is a fan of Saul Kripke
$C(x)$	x is in favor of constant domains
a	Aravik

Consider the following set of default rules:

$$D = \left\{ \delta_1 = \frac{P(x) \wedge B(x) : \neg C(x)}{\neg C(x)}, \qquad \delta_2 = \frac{\neg K(x) : C(x)}{C(x)}, \qquad \delta_3 = \frac{P(x) : B(x)}{B(x)} \right\},$$

and initial set of facts:

$$W = \{P(a), \forall x(\neg B(x) \vee \neg K(x))\}.$$

This exercise is about the default theory $T = (W, D)$; so you only need to apply the defaults to the relevant constant a.

(a) Of each of the following sequences, state whether it is a *process*; and if so, whether or not the process is *closed*, and whether or not it is *successful*. Briefly explain your answers.

   i. the empty sequence ( )
   ii. (δ_2)
   iii. (δ_3, δ_2)
   iv. $(\delta_3, \delta_2, \delta_1)$

(b) Draw the process tree of the default theory (W, D).

(c) What are the extensions of (W, D)?

Bonus; 10 pt Consider the language of FDE, containing the connectives $\neg, \vee, \wedge$ (and not containing $\supset$). Does the following statement hold for all wffs A, B in that language?

"If $A \vdash_{K_3} B$ and $A \vdash_{LP} B$, then $A \vdash_{FDE} B$"

If yes, please explain exactly why the statement holds for all wffs A, B in the language of FDE. If no, please provide a pair of wffs A, B and show in detail why that pair forms a counterexample to the statement.